AP Calculus AB

Derivatives of Inverse Trig

1) $y = \arccos(x^2)$

$$y' = \frac{-d[x^2]}{\sqrt{1-[x^2]^2}}$$

$$= \frac{-2x}{\sqrt{1-x^4}}$$

2) $y = \arcsin\sqrt{2t}$

$$y' = \frac{d[(2t)^{1/2}]}{\sqrt{1-[(2t)^{1/2}]^2}}$$

$$y' = \frac{\frac{1}{2}(2t)^{-1/2}\cdot 2}{\sqrt{1-2t}}$$

$$y' = \frac{1}{\sqrt{2t}\sqrt{1-2t}}$$

3) $y = \arcsin\frac{3}{t^2}$

$y = \arcsin(3t^{-2})$

$$y' = \frac{-6t^{-3}}{\sqrt{1-(3t^{-2})^2}}$$

$$y' = \frac{-6}{t^3\sqrt{1-9t^{-4}}}$$

$$y' = \frac{-6}{t^3\sqrt{1-\frac{9}{t^4}}}$$

$$y' = \frac{-6}{t^3\sqrt{\frac{t^4}{t^4}-\frac{9}{t^4}}}$$

$$y' = \frac{-6}{t^3\frac{\sqrt{t^4-9}}{t^2}}$$

$$y' = \frac{-6}{t\sqrt{t^4-9}}$$

*4) $y = x\arcsin x + (1-x^2)^{1/2}$

$$y' = x\cdot\frac{1}{\sqrt{1-x^2}} + \arcsin x + \frac{1}{2}(1-x^2)^{-1/2}\cdot(-2x)$$

$$y' = \frac{x}{\sqrt{1-x^2}} + \arcsin x - \frac{x}{\sqrt{1-x^2}}$$

$$y' = \arcsin x$$

5) $y = \operatorname{arcsec} 5s$

$$y' = \frac{d[5s]}{|5s|\sqrt{[5s]^2-1}}$$

$$y' = \frac{5}{|5s|\sqrt{25s^2-1}}$$

6) $y = \arctan\sqrt{t-1}$

$$y' = \frac{\frac{1}{2}(t-1)^{-1/2}}{1+(\sqrt{t-1})^2} = \cdot\frac{\frac{1}{2}(t-1)^{-1/2}}{1+t-1} = \frac{\frac{1}{2}(t-1)^{-1/2}}{t}$$

$$y' = \frac{1}{2t\sqrt{t-1}}$$

7) $\frac{d}{dx}\arcsin\left(\frac{1}{2}x\right)$

$$\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}x\right)^2}}$$

$$\frac{\frac{1}{2}}{\sqrt{1-\frac{1}{4}x^2}}$$

$$\frac{\frac{1}{2}}{\sqrt{\frac{4}{4}-\frac{x^2}{4}}}$$

$$\frac{1}{2\sqrt{\frac{4-x^2}{4}}}$$

$$\frac{1}{\sqrt{4-x^2}}$$ (E)

8) $y = 3 - 7x^3 + 3x^7$

$y' = -21x^2 + 21x^6$

9) $y = \frac{2x+1}{2x-1}$

$$y' = \frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2}$$

$$y' = \frac{4x-2-4x-2}{(2x-1)^2} = \frac{-2}{(2x-1)^2}$$

10) $y = \cot(2t^{-1})$

$y' = 2t^{-2}\csc^2(2t^{-1})$

11) $y = x(2x+1)^{1/2}$

$$y' = x\left[\frac{1}{2}(2x+1)^{-1/2}\cdot 2\right] + (2x+1)^{1/2}$$

$$= \frac{x}{\sqrt{2x+1}} + \sqrt{2x+1}\cdot\frac{\sqrt{2x+1}}{\sqrt{2x+1}}$$

$$= \frac{x}{\sqrt{2x+1}} + \frac{2x+1}{\sqrt{2x+1}}$$

$$= \frac{3x+1}{\sqrt{2x+1}}$$

12) $r = \tan^2(3-\theta^2) = [\tan(3-\theta^2)]^2$

$r' = 2\tan(3-\theta^2)\cdot\sec^2(3-\theta^2)(-2\theta)$

13) $y = \ln\sqrt{x}$

$y = \ln x^{1/2}$

$y = \frac{1}{2}\ln x$

$y' = \frac{1}{2}\left(\frac{1}{x}\right) = \frac{1}{2x}$

14) $y = xe^{-x}$

$y' = x(-e^{-x}) + e^{-x}$

15) $y = \ln[\sin x]$

$$y' = \frac{d[\sin x]}{[\sin x]}$$

$$y' = \frac{\cos x}{\sin x} = \cot x$$